Mr. Michael T. Davis Pre-Calculus	2.9 Determining Equations of Vertical & Horizontal Asymptotes from Equations and Identifying Holes (1) February 2, 2016

Name: Solutions

Directions: Identify equations for all vertical and horizontal asymptotes. Also, identify any value of x for which the graph has a hole (a removable discontinuity)

Identify x-intercepts y-intercepts

1. $f(x)=\frac{1}{x}$ numerator degree 0 H.A. $y=0$
denominator degree 1
restriction $x \neq 0$ V.A. $x=0$
No x-intercept since the numerator cannot be zero
No y-intercept since x cannot be zero

2. $f(x)=\frac{2}{x-3}$ Numerator degree 0 H.A. $y=0$
denominator degree 1
restriction $x \neq 3$ V.A. $x=3$
No x-intercept since the numerator cannot be zero
The y-intercept is when $x=0$ therefore $y=\frac{-2}{3}$
$(0,\frac{-2}{3})$

3. $f(x)=\frac{3}{x+2}$ Numerator degree 0 H.A. $y=0$
denominator degree 1
restriction $x \neq -2$ V.A. $x=-2$
No x-intercept since the numerator cannot be zero
y-intercept is when $x=0$ therefore $(0,1\frac{1}{2})$

4. $f(x)=\frac{4}{(x-1)(x+3)}$ Numerator degree 0 H.A. $y=0$
denominator degree 2
restrictions $x \neq 1$ and $x \neq -3$ V.A. $x=1$ and $x=-3$
y-intercept is when $x=0$ therefore $(0,\frac{-4}{3})$
No x-intercept since the numerator cannot be zero

5. $f(x)=\frac{x}{x-5}$ numerator degree 1 H.A. $y=\frac{1}{1}$ $y=1$
denominator degree 1
restrictions $x \neq 5$ V.A. $x=5$
x-intercept comes from y being zero or $f(x)=0$
therefore $0=\frac{x}{x-5}$ which is when $x=0$ so $(0,0)$
y-intercept is when $x=0$ therefore $(0,0)$

6. $f(x)=\dfrac{x-4}{(x-4)(x+1)}$ degree 1 / degree 2 H.A. $y=0$

restrictions $x \neq 4$, $x=-1$

Hole at $x=4$ V.A. $x=-1$

No x-intercept

y-intercept when $x=0$, therefore $f(0)=1$ $(0,1)$

7. $f(x)=\dfrac{x^2-4}{(x-4)(x+1)}$ degree 2 / degree 2 H.A. $y=\frac{1}{1}=1$ $y=1$

restrictions $x \neq 4$ & $x \neq -1$ V.A. $x=4$ and $x=-1$

x-intercept from Numerator being zero at $x=2$ & $x=-2$

x-intercepts $(2,0)$ and $(-2,0)$

y-intercept from $f(0)=\dfrac{-4}{(-4)(1)}=1$ $(0,1)$

8. $f(x)=\dfrac{2x^2-4}{(x-4)(x+1)}$ degree 2 / degree 2 H.A. $y=\frac{2}{1}=2$ $y=2$

restrictions $x \neq 4$ $x \neq -1$ V.A. $x=4$ & $x=-1$

x-intercepts: $2x^2-4=0$ $x^2-2=0$ $x=\pm\sqrt{2}$ $(\sqrt{2},0)$ & $(-\sqrt{2},0)$

y-intercept: $f(0)=1$ $(0,1)$

9. $f(x)=\dfrac{1}{x^2+4x-12}$ degree 0 / degree 2 H.A. $y=0$ $f(x)=\dfrac{1}{(x+6)(x-2)}$

V.A. $x=-6$ & $x=2$

x-intercept: none

y-intercept: $f(0)=\frac{-1}{12}$ $(0,-\frac{1}{12})$

10. $f(x)=\dfrac{x+6}{x^2+4x-12}$ degree 1 / degree 2 H.A. $y=0$ $f(x)=\dfrac{x+6}{(x+6)(x-2)}$

Hole at $x=-6$ V.A. $x=2$

No x-intercept

y-intercept: $f(0)=-\frac{1}{2}$ $(0,-\frac{1}{2})$

11. $f(x)=\dfrac{2}{x^2-9}$ H.A. $y=0$ $f(x)=\dfrac{1}{(x-3)(x+3)}$

V.A. $x=3$ & $x=-3$

No x-intercept

y-intercept: $(0,-\frac{2}{9})$

12. $f(x)=\dfrac{5}{x^2+9}$ H.A. $y=0$ No V.A. since $x^2+9\neq 0$

No x-intercept

y-intercept: $f(0)=\frac{5}{9}$ $\left(0,\frac{5}{9}\right)$

13. $f(x)=\dfrac{3x}{x^2+9}$ H.A. $y=0$ No V.A. since $x^2+9\neq 0$

x-intercept when $f(x)=0$ $0=\dfrac{3x}{x^2+9}$ $x=0$ $(0,0)$

y-intercept: $f(0)=0$ $(0,0)$

14. $f(x)=\dfrac{3x^2}{x^2+9}$ degree 2 / degree 2 H.A. $y=\frac{3}{1}$ $y=3$

No V.A.

x-intercept $(0,0)$

y-intercept $(0,0)$

15. $f(x)=\dfrac{8x^2}{2x^2+9}$ H.A. $y=\frac{8}{2}=4$ $y=4$ No V.A.

x-intercept $(0,0)$

y-intercept $(0,0)$

16. $f(x)=\dfrac{x+1}{x^3-3x^2-4x}=\dfrac{x+1}{x(x-4)(x+1)}$ H.A. $y=0$

V.A. $x=4$ & $x=0$ $\boxed{\text{Hole at } x=-1}$

No x-intercept

y-intercept: $f(0)=$ undefined so no y-intercept

17. $f(x)=\dfrac{x^2-4x-5}{x^3-3x^2-4x}=\dfrac{(x-5)(x+1)}{x(x-4)(x+1)}$ H.A. $y=0$

V.A. $x=4$ & $x=0$ $\boxed{\text{Hole at } x=-1}$

x-intercept $(5,0)$

y-intercept: $f(0)=$ undefined so no y-intercept